Propagation of Gaussian beams in a nonlinear saturable medium

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A variational procedure is implemented to investigate the propagation of cw Gaussian beams in a saturable medium with and without loss. The saturation is modeled by taking into account higher-order terms in the intensity-dependent refractive index. In the lossless case, exact analytical expressions for the behavior of the spot width are determined, along with the condition for steady-state propagation. Gaussian beams under dissipation are also considered, and the results agree with recent numerical simulations.

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I. INTRODUCTION

Solitary wave solutions have been known to exist in a variety of nonlinear, dispersive media for many years. In the context of optical communications, Hasegawa and Tappert [1] first made the important observation that a pulse propagating in an optical fiber with Kerr-law nonlinearity can form an envelope soliton. This offered the potential for undistorted pulse transmission over very long distances, but it was several years before improvements in fiber and source technology allowed Mollenauer, Stolen, and Gordon [2] to experimentally demonstrate soliton propagation in fibers. Since that time, spectacular advances have been made in the development of experimental, high-capacity, long-distance transmission systems using solitons [3].

Just as a balance between self-phase-modulation and group-velocity dispersion can lead to the formation of temporal solitons in single-mode fibers, it is also possible to have the analogous spatial soliton, where diffraction and self-focusing can compensate for each other [4]. It is well known that optical beams propagating in a Kerr-law medium with two transverse dimensions are unstable at the critical power where self-focusing occurs [5]. Hence, for a spatial soliton to be realized, diffraction must be limited to one transverse direction. Spatial optical solitons have recently been observed [6] in single-mode planar waveguides, where confinement is provided in one transverse direction by a linear refractive-index difference and in the orthogonal direction by self-trapping due to an intensity-dependent refractive-index variation.

A soliton is a particular solution of the wave equation. Since analytical solutions are known for only a few cases, investigations into the properties of solitons are normally performed numerically using such approaches as the beam propagation method (BPM) [7]. However, it is often desirable to have an analytical model describing the dynamics of pulse propagation in a fiber. To this end, various approximation methods have been developed, such as perturbation [8], equivalent particle [9], and variational methods [10].

Recently, much attention has been given to the variational approach [11]. It is able to provide succinct information about the various parameters that characterize the beam that could be used for the design of waveguides. Numerical results indicate that the variational approximation is a good one [12]. A variational treatment of the wave equation has been shown to be relatively accurate for a large range of incident powers of the incoming wave.

A variational approach was employed by Anderson [13] in order to describe the main characteristics of the temporal soliton as determined by the cubic nonlinear Schrödinger equation (NLSE). These results are equally applicable to the spatial solitons. In real materials, the nonlinearity will not be of pure Kerr-law form, but will saturate. In this paper, we extend the analysis of Anderson to model saturation effects by including a quintic term in the NLSE.

The inclusion of a fifth-order nonlinearity is not merely a mathematical generalization of the cubic NLSE, but arises physically from the retention of higher-order terms in the nonlinear polarization tensor. This leads to a refractive index of the form

$$n = n_0 + i\eta + n_2|E|^2 - n_4|E|^4 , \tag{1}$$

with E being the electric field transmitted in the waveguide, n_0 the linear refractive index of the medium, η the medium loss, and n_2 and n_4 the third- and fifth-order nonlinear coefficients. Thus an intense beam would be required to make $n_4|E|^4$ of the same order as $n_2|E|^2$. In spite of this, there appears to be both theoretical and experimental interest in fifth-order nonlinear effects [14–17].

Exact solutions for the *TE* polarized nonlinear waves guided by an interface between a linear medium and a saturable, self-focusing medium modeled by Eq. (1) have been published along with the numerical investigation of their stability [14]. Also, the interaction of solitons taking into account the fifth-order nonlinearity in the refractive index was recently studied numerically [15]. On the experimental side, fifth-harmonic generation in Ne and Ar with laser pulses focused to intensities in the $10^{13}-10^{15}$ W/cm range have been performed [16], as was a determination of the nonlinear fifth-order susceptibility of InSb at 10.6 μm [17].

The purpose of this work is twofold: first, to understand the effect of the fifth-order nonlinearity compared to the cubic nonlinearity alone; and, second, to obtain explicit solutions for the dynamical behavior of the beamwidth and other parameters, which eluded previous authors [18].

II. THEORY

Losses are, of course, inevitable in real materials. The Kramers-Krönig relation dictates that at least linear absorption must accompany nonlinear refraction. Thus we consider the wave equation (for cw beams)

$$2i\beta\left[\frac{\partial E}{\partial z}+\gamma E\right]+\frac{\partial^2 E}{\partial x^2}-[\beta^2-k_0^2n^2(x,\alpha|E|^2)]E=0\ , \tag{2}$$

where $n^2(x,\alpha|E|^2)=n_0^2+\alpha_1|E|^2-\alpha_2|E|^4$, and $\gamma=k_0^2\eta^2/2\beta$. Equation (2) will be solved variationally by assuming a trial function of the form

$$E(z,x)=A(z)\exp[-x^2/2a^2(z)+ib(z)x^2]\ . \tag{3}$$

If we let $\beta^2=k_0^2n_0^2$ and make the amplitude transformation $E=\psi e^{-\gamma z}$, then the above equation can be written as

$$2i\beta\frac{\partial\psi}{\partial z}+\frac{\partial^2\psi}{\partial x^2}+k_0^2\alpha_1e^{-2\gamma z}|\psi|^2\psi-k_0^2\alpha_2e^{-4\gamma z}|\psi|^4\psi=0\ . \tag{4}$$

The wave equation can be reformulated as a variational equation according to the variational principle

$$\delta\int\int\mathcal{L}(\psi,\psi^*,\psi_x,\psi_x^*,\psi_z,\psi_z^*)dz\,dx=0\ , \tag{5}$$

where

$$\mathcal{L}=i\beta\left[\psi\frac{\partial\psi^*}{\partial z}-\psi^*\frac{\partial\psi}{\partial z}\right]+\left|\frac{\partial\psi}{\partial x}\right|^2-\tfrac{1}{2}k_0^2\alpha_1e^{-2\gamma z}|\psi|^4+\tfrac{1}{3}k_0^2\alpha_2e^{-4\gamma z}|\psi|^6\ .$$

We assume initially that the input cw wave is a Gaussian,

$$\psi(0,x)=B_0e^{-x^2/2a_0^2}\ , \tag{6}$$

where B_0 and a_0 are the initial amplitude and spot width, respectively. Subsequent evolution in the medium is then assumed to be specified by

$$\psi(z,x)=B(z)\exp[-x^2/2a^2(z)+ib(z)x^2]\ , \tag{7}$$

where the amplitude $B(z)$, spot width $a(z)$, and inverse wave front curvature $b(z)$ are all allowed to vary with propagation distance. On account of the above amplitude transformation, we have

$$A(z)=B(z)e^{-\gamma z}\ , \tag{8}$$

and $B(z)=|B(z)|e^{i\phi(z)}$, with $\phi(z)$ being the longitudinal phase. By substituting Eq. (7) into the variational principle and integrating over the transverse coordinate x, we obtain the so-called "reduced" variational principle given by

$$\delta\int L_G dz=0\ , \tag{9}$$

where

$$L_G=\frac{i\beta}{2}\left[B\frac{dB^*}{dz}-B^*\frac{dB}{dz}\right]\sqrt{\pi}+|B|^2\left[\beta\frac{db}{dz}+2b^2\right]\frac{a^2\sqrt{\pi}}{2}+\frac{|B|^2\sqrt{\pi}}{4a}-\frac{k_0^2\alpha_1e^{-2\gamma z}\sqrt{\pi}}{4\sqrt{2}}|B|^4a+\frac{k_0^2\alpha_2e^{-4\gamma z}\sqrt{\pi}}{6\sqrt{3}}|B|^6a\ . \tag{10}$$

By deriving the Euler-Lagrange equations for B, B^*, a, and b, we obtain after some algebra the following set of coupled ordinary differential equations:

$$b(z)=\frac{\beta}{2a}\frac{da}{dz}\ , \tag{11a}$$

$$\beta^2\frac{d^2a}{dz^2}=\frac{1}{a^3}-\frac{k_0^2\alpha_1e^{-2\gamma z}}{2\sqrt{2}}\frac{I}{a^2}+\frac{2k_0^2\alpha_2e^{-4\gamma z}}{3\sqrt{3}}\frac{I^2}{a^3}\ , \tag{11b}$$

$$2\beta\frac{d\phi}{dz}=-\frac{1}{a^2}+\frac{5\sqrt{2}k_0^2\alpha_1e^{-2\gamma z}}{8}\frac{I}{a}-\frac{4k_0^2\alpha_2e^{-4\gamma z}}{3\sqrt{3}}\frac{I^2}{a^2}\ , \tag{11c}$$

and $|B(z)|^2a=|B_0|^2a_0=I$ is a constant of the motion of Eq. (4). This system of equations has no analytic solutions. However, we initially look at the lossless case, by setting $\gamma=0$, and show that in that case, an analytic solution can be found.

III. BEAM DYNAMICS IN A LOSSLESS MEDIUM

In the case of lossless Gaussian propagation in the saturable nonlinear medium, we set $\gamma=0$. Recent attempts to determine analytic solutions for the spot width of temporal Gaussian solitons in a saturable nonlinear medium via the invariants of the nonlinear wave equation have failed [18]. It was pointed out [19] that the ansatz employed in [18] did not conserve power, which consequently led to unphysical results such as the prediction of pulse width collapse at a finite distance of propagation.

We will now show that the above system of differential equations has an analytic solution. To do this, we focus our attention on Eq. (11b) since, once $a(z)$ is determined, $b(z)$ and $\phi(z)$ can also be found, and thus a knowledge of the dynamical behavior of the cw beam can be ascertained.

We normalize the spot width to the initial spot width by letting $y(z)=a(z)/a_0$, so that Eq. (11b) becomes

$$\beta^2\frac{d^2y}{dz^2}=\frac{2\mu}{y^3}-\frac{\nu}{y^2}+\frac{2\lambda}{y^3}\ , \tag{12}$$

with

$$\mu=\frac{1}{2a_0^4}\ ,\quad \nu=\frac{k_0^2\alpha_1|B_0|^2}{2\sqrt{2}a_0^2}\ ,\quad \lambda=\frac{k_0^2\alpha_2|B_0|^4}{3\sqrt{3}a_0^2}\ .$$

For the lossless case, it is not difficult to show, upon integration, that the spot width satisfies the dynamical equation

$$\frac{\beta^2}{2}\left[\frac{dy}{dz}\right]^2+\Pi(y)=0\ , \tag{13}$$

where

$$\Pi(y)=\frac{\mu}{y^2}-\frac{\nu}{y}+\frac{\lambda}{y^2}-(\mu-\nu+\lambda)\ . \tag{14}$$

The behavior of the spot width is controlled by the nature of the potential function $\Pi(y)$, as the equation can be interpreted as a "point mass" under the influence of the potential. It is clear that as $y\to 0^+$, $\Pi(y)\to\infty$, and that as $y\to\infty$, $\Pi(y)\to-(\mu-\nu+\lambda)$. We note also that $\Pi(1)=0$. We are able to find solutions for the spot width by looking at the integral

$$\pm\frac{\sqrt{2}z}{\beta}=\int_1^y\frac{dy}{\sqrt{-\Pi(y)}}\ , \tag{15}$$

and the type of solution will depend on the (μ,ν,λ) parameter space we choose.

In Fig. 1, the potential function $\Pi(y)$ is plotted for different regions of the (μ,ν,λ) parameter space. The nature of the potential function allows us to subdivide the parameter space as follows: (a) $\mu+\lambda-\nu\geq 0$; (b) $\mu+\lambda-\nu<0$, with two subregions (i) $\mu+\lambda<\nu<2(\mu+\lambda)$, and (ii) $\nu>2(\mu+\lambda)$. The dynamics of the beam in each of these regions will now be investigated.

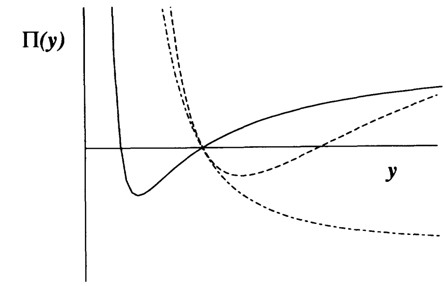

FIG. 1. Qualitative plot of the potential function $\Pi(y)$ in the (μ,ν,λ) parameter space: $\nu\leq\mu+\lambda$ (dashed-dotted curve); $\mu+\lambda<\nu<2(\mu+\lambda)$ (dashed curve); $\nu>2(\mu+\lambda)$ (solid curve).

(a) $\mu+\lambda-\nu\geq 0$. For $\nu\leq\mu+\lambda$, the spot width once released at $y=1$ will monotonically increase. The combined effect of linear diffraction and the fifth-order nonlinearity overcome the cubic nonlinearity, so no stable solution exists.

In the limiting case $\nu=\mu+\lambda$, the solution is given by

$$\frac{3\sqrt{2(\mu+\lambda)}}{2\beta}z=(y+2)\sqrt{y-1}\ , \tag{16}$$

and quite clearly the Gaussian beam is diffracting as the spot width y increases with increasing z. For the more general case, we let $\zeta=\nu/(\mu+\lambda)$ and find solutions given by

$$\frac{\sqrt{2(\mu+\lambda)}}{\beta}z=\left[\frac{(y-1)[y+1/(1-\zeta)]}{1-\zeta}\right]^{1/2}-\frac{\zeta}{2(1-\zeta)^{3/2}}\ln\left[\frac{2(1-\zeta)\{(y-1)[y+1/(1-\zeta)]\}^{1/2}+2(1-\zeta)y+\zeta}{2-\zeta}\right]\ . \tag{17}$$

For very large y, the spot width increases according to

$$y\sim\frac{\sqrt{2(\mu+\lambda)(1-\zeta)}}{\beta}z\ . \tag{18}$$

(b) $\mu+\lambda-\nu<0$. In this region, we see that the potential function has a minimum [i.e., $\Pi'(y)=0$] at $y_e=2(\mu+\lambda)/\nu$, and zeros at $y_0=1$, $y_1=-(\mu+\lambda)/(\mu-\nu+\lambda)$. We further note $\Pi(y_e)=-[2(\mu+\lambda)-\nu]^2/4(\mu+\lambda)\leq 0$, as $\mu>0$ and $\lambda\geq 0$.

(i) $\mu+\lambda<\nu<2(\mu+\lambda)$. In this region, the potential function has two real roots with $y_1>y_0$. Here, a beam with $y_0=1$ would initially diffract until it attains the largest possible value at y_1, at which stage self-focusing effects become dominant and the spot width decreases, returning to its minimum value y_0. As the spot width executes a homoclinic orbit, the resultant behavior is oscillatory.

The solution for the spot width in this region is given by

$$\frac{\sqrt{2(\mu+\lambda)}}{\beta}z=-\left[\frac{(1-y)[y-1/(\zeta-1)]}{\zeta-1}\right]^{1/2}+\frac{\zeta}{2(\zeta-1)^{3/2}}\left[\sin^{-1}\frac{2(\zeta-1)y-\zeta}{2-\zeta}+\frac{\pi}{2}\right]\ , \tag{19}$$

and the period of the oscillation is given by z_p, where

$$\frac{\sqrt{2(\mu+\lambda)}}{\beta}z_p=\frac{\pi\zeta}{(\zeta-1)^{3/2}}\ . \tag{20}$$

(ii) $\nu>2(\mu+\lambda)$. Here $y_1<y_0$, and thus the spot width initially decreases until it attains the minimum value y_1, at which point it becomes sufficiently small so that diffractive forces dominate and the spot width increases again until it reaches maximum value y_0. Once again, the behavior in this region of the parameter space is oscillatory.

The solution for the spot width is given by

$$\frac{\sqrt{2(\mu+\lambda)}}{\beta}z=\left[\frac{(1-y)[y-1/(\zeta-1)]}{\zeta-1}\right]^{1/2}+\frac{\zeta}{2(\zeta-1)^{3/2}}\left[\sin^{-1}\frac{2(\zeta-1)y-\zeta}{2-\zeta}+\frac{\pi}{2}\right], \quad (21)$$

with the period of oscillation given by Eq. (20).

(iii) $\nu=2(\mu+\lambda)$. For the special case in which $\nu=2(\mu+\lambda)$, we see that $y_e=1$ and that $\Pi'(1)=\Pi(y_e)=0$. The potential well has degenerated into a single point and a particle released at this point will remain there. This translates into a beam propagating undistorted. There is an exact balance between the competitive forces of diffraction and self-focusing. The steady-state spot width is given by

$$\frac{1}{a_0}=k_0\left[\frac{\alpha_1}{2\sqrt{2}}-\frac{2\alpha_2}{3\sqrt{3}}|B_0|^2\right]^{1/2}|B_0| . \quad (22)$$

In the case of a cubic nonlinearity, we set $\alpha_2=0$ and obtain the value

$$\frac{1}{a_0}=k_0\left[\frac{\alpha_1}{2\sqrt{2}}\right]^{1/2}|B_0| , \quad (23)$$

which agrees with [13]. So we see that the steady-state spot width is larger in the saturable medium. The corresponding phase shift is given by

$$2\beta\phi(z)=k_0^2\left[\frac{3\sqrt{2}}{8}\alpha_1|B_0|^2-\frac{2\sqrt{3}}{9}\alpha_2|B_0|^4\right]z . \quad (24)$$

Once again, for $\alpha_2=0$ we obtain the well-known result for the cubic nonlinearity [13]

$$2\beta\phi_0=k_0^2\left[\frac{3\sqrt{2}}{8}\alpha_1|B_0|^2\right]z . \quad (25)$$

For sufficiently high intensities, Eq. (24) gives the possibility of $\phi(z)=0$, which corresponds to a localized stationary wave.

Figure 2 illustrates the beam spot width and longitudinal phase variations as functions of propagation distance for the different regions of the (μ,ν,λ) parameter space. We have already discussed the spot width at length, so here we shall consider the longitudinal phase shift $\phi(z)$. With $\gamma=0$, we see from Eq. (11c) that the phase is dependent on z through $a(z)$. So if $a(z)$ is oscillatory, then ϕ_z' will oscillate as well. We note that ϕ_z' increases as $a(z)$ increases, and that in the linear case $\phi_z'<0$, for all z. For the nonlinear lossless steady-state problem, ϕ_z' may be interpreted as an eigenvalue of Eq. (4), and ϕ_z' is then the propagation constant of the beam as seen from Eq. (24). We also note that for the steady-state problem, ϕ_z' is a positive constant for all powers, indicating that the effective index of the induced waveguide is greater than n_0.

We are now in a position to investigate the stability of cw Gaussian beams in this (μ,ν,λ) parameter space. According to Eq. (13), a perturbation from the equilibrium

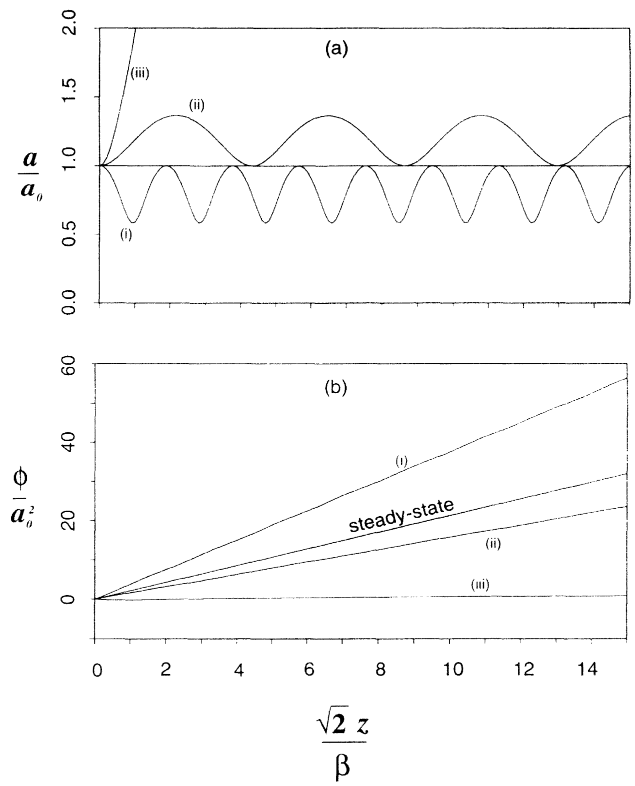

FIG. 2. Propagation dependence of normalized (a) spot width and (b) longitudinal phase in the(μ,ν,λ) parameter space: (i) oscillatory self-focusing; (ii) oscillatory diffracting (iii) monotonically diffracting. The initial beam width is a_0.

makes $d^2y/dz^2\neq0$. If a variation in the beam parameters is such that it tends to re-establish the delicate balance between diffraction and self-focusing, the beam is said to be stable; otherwise it is unstable.

Stability may be determined by performing a Taylor expansion of the potential about the equilibrium point $y\approx y_e$, and linearizing the dynamical equation to find

$$\beta^2\frac{d^2}{dz^2}(y-y_e)+\Pi''(y_e)(y-y_e)=0 , \quad (26)$$

with $\Pi''(y_e)=\nu^4/8(\mu+\lambda)^3$. The quantity $\Pi''(y_e)$ is always positive, indicating stable equilibrium. The spot width will now oscillate with the period given by

$$\frac{\sqrt{2(\mu+\lambda)}}{\beta}z_p=\frac{8\pi}{\zeta^2} . \quad (27)$$

These results agree with the well-known fact that solitons are stable in a nonlinear medium with one transverse dimension; hence their potential use in all-optical devices.

In Fig. 3, we have summarized the previous discussion in order to get a clear picture of the beam behavior in the various regions. The solid line depicted by $\nu=\mu+\lambda$ establishes the region between stability and diffraction of the cw Gaussian beam. We will also gain further understanding when we discuss the effect of dissipation on the beam in the next section.

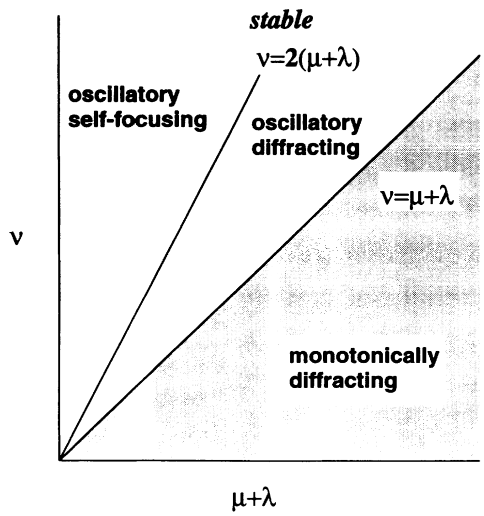

FIG. 3. Regions of stability and instability in the $(\nu,\mu+\lambda)$ plane.

IV. BEAM DYNAMICS IN A LOSSY MEDIUM

In this section, we investigate the effect of fifth-order nonlinearity in the presence of loss. As mentioned earlier, if $\gamma\neq 0$ then no exact solutions exist for the set of coupled equations given by Eqs. (11a)–(11c). We look first at lossy linear and Kerr media.

A. Linear medium

From the dynamical equation, we obtain the linear limit by allowing $\nu=0$ ($\alpha_1=0$) and $\lambda=0$ ($\alpha_2=0$). For this case, we are able to find the exact solution given by

$$E(z,x)=Be^{-\gamma z}\left[1-i\frac{z}{\beta a_0^2}\right]^{1/2}\exp\left[\frac{-x^2/2a_0^2}{1-iz/\beta a_0^2}\right], \quad (28)$$

which is the well-known result for a Gaussian propagating in a diffractive lossy medium. We particularly note that the amplitude of the linear wave is given by

$$A(z)=Be^{-\gamma z}. \quad (29)$$

B. Kerr medium

For $\lambda=0$ ($\alpha_2=0$), the dynamical equations reduce to those used by Anderson [20], to investigate the properties of temporal solitons in the presence of dissipation. The spot width is given by

$$\beta^2\frac{d^2y}{dz^2}=\frac{2\mu}{y^3}-\frac{\nu e^{-2\gamma z}}{y^2}. \quad (30)$$

Normally, numerical means are required to solve this equation in order to investigate the nature of the spot width.

It has been shown using a perturbative procedure [21] that a soliton satisfying the NLSE preserves its soliton character by adiabatically adjusting width to its decreasing intensity. It is found that the spot width and amplitude satisfy

$$y(z)\propto e^{2\gamma z}, \quad |A(z)|\propto e^{-2\gamma z}. \quad (31)$$

These results would imply that eventually the beam width should exceed that of a linear spreading beam. Numerical [22] results suggest that the above adiabatic solution is only valid for small propagation distances.

For $\gamma z\ll 1$, a good approximation to the initial evolution of the spot width [20] is obtained by assuming that $d^2y/dz^2\approx 0$ in Eq. (30). This yields

$$y(z)\approx\frac{2\mu}{\nu}e^{2\gamma z}, \quad |A(z)|\approx\left[\frac{\nu}{2\mu}\right]^{1/2}Ie^{-2\gamma z}. \quad (32)$$

Hence we are faithfully able to reproduce the adiabatic solutions and confirm their validity for small propagation distances.

More recent numerical simulations [23] suggest that the amplitude decreases initially as $Be^{-2\gamma z}$, but finally as $Be^{-\gamma z}$. The deviation occurs when the amplitude diminishes sufficiently so that the nonlinear and diffractive effects no longer balance each other. The spot width does not become large in proportion to the decrease in the amplitude. In this case, the wave loses its soliton property and is considered to be merely a linear wave. In such a situation, the amplitude decreases but the spot width remains almost constant.

For $\gamma z\gg 1$, the importance of the nonlinearity vanishes, and the asymptotic evolution of the spot width is governed by

$$\beta^2\frac{d^2y}{dz^2}=\frac{2\mu}{y^3}. \quad (33)$$

This is the equation that characterizes the pure diffraction of a Gaussian beam, but it needs to be interpreted correctly. It applies after some evolution of the wave, where diffraction dominates. In this regime, the spot width $y(z)$ would have expanded considerably and, noting that diffractive forces vary as y^{-1}, then the force acting on it is small. This implies that there would be only a gradual further increase with propagation distance, which is accompanied by a decrease in amplitude as for a linear wave. This is consistent with the numerical findings of [23].

C. Saturable medium

The equation characterizing the dynamics of the spot width in a saturable medium is given by

$$\beta^2\frac{d^2y}{dz^2}=\frac{2\mu}{y^3}-\frac{\nu e^{-2\gamma z}}{y^2}+\frac{2\lambda e^{-4\gamma z}}{y^3}. \quad (34)$$

For $\gamma z\ll 1$, we can look at approximate solutions for the early evolution of the beam by letting $d^2y/dz^2\approx 0$ in Eq. (34). We then find in the initial stages of evolution that the spot width increases according to

$$y(z)\approx\frac{2\mu}{\nu}e^{2\gamma z}+\frac{2\lambda}{\nu}e^{-2\gamma z}. \quad (35)$$

This rate of increase is slightly larger than that of the beam propagating in a Kerr medium, as the fifth-order nonlinearity aids the diffractive process ($\lambda > 0$). For large propagation distances, the nonlinearity is insignificant and we recover the asymptotic equation describing the diffraction of a Gaussian beam propagating in a linear lossy medium.

Numerical solutions of Eq. (34) have recently been published for the temporal Gaussian soliton [24]. It was found that the fifth-order nonlinearity considerably modified the beam propagation. These numerical results suggest the frequency at which reshaping is required to achieve a distortionless propagation for a Gaussian beam in a saturable medium perturbed by dissipation reduced by a factor ~ 3 compared with the cubic nonlinearity.

The propagation of cw beams in waveguides is vastly different from that in fibers for telecommunications. In fibers, the attenuation is $\sim 10^{-9}$ μm^{-1}, whereas waveguides for the potential use in all-optical devices typically have loss [25] given by $\gamma \sim 10^{-3} - 10^{-5}$ μm^{-1}. Hence the wave is significantly attenuated in waveguides. We investigate the dynamics of the beam propagation in a lossy waveguide by numerically solving the system of Equations (11a)–(11c). The results of the numerical analysis are depicted in Figs. 4 and 5. We have considered the situations where the spot width undergoes initial decompression and also initial compression.

We have examined two configurations, namely $\sqrt{2}\beta\gamma = 0.01$ μm^{-2} and $\sqrt{2}\beta\gamma = 0.025$ μm^{-2}. Numerical results show that the spot width of a cw Gaussian beam initially oscillates, passing through a series of maxima and minima before finally diffracting. The presence of the attenuation reduces the number of oscillations. The critical distance z_{cr}, at which the beam has lost it solitonic character, is $\sqrt{2}z_{\text{cr}}/\beta \approx 100$ μm^2 and $\sqrt{2}z_{\text{cr}}/\beta \approx 40$ μm^2 for the lower and greater loss, respectively. This is clearly seen from Fig. 4. This critical distance is increased when the beam initially self-focuses (see Fig. 5).

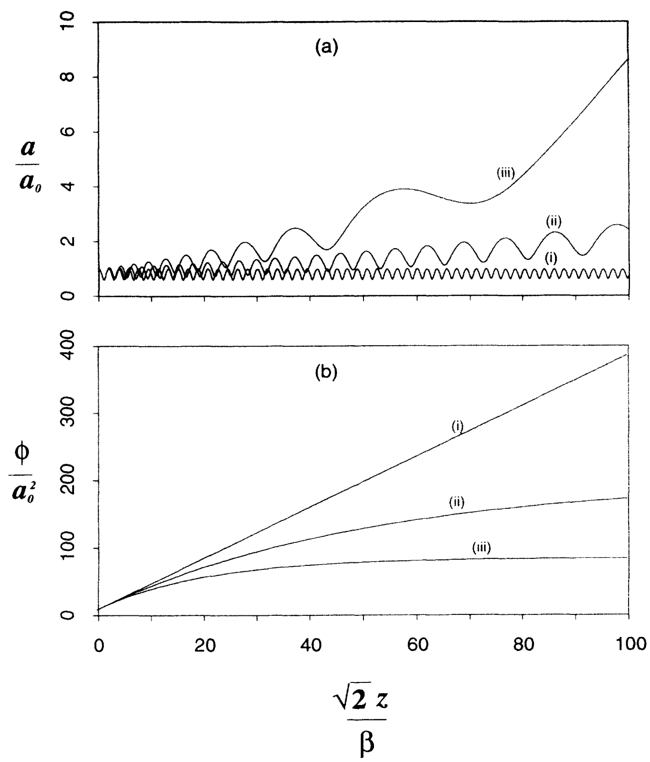

FIG. 5. As for Fig. 4 but with $\nu > 2(\mu+\lambda)$.

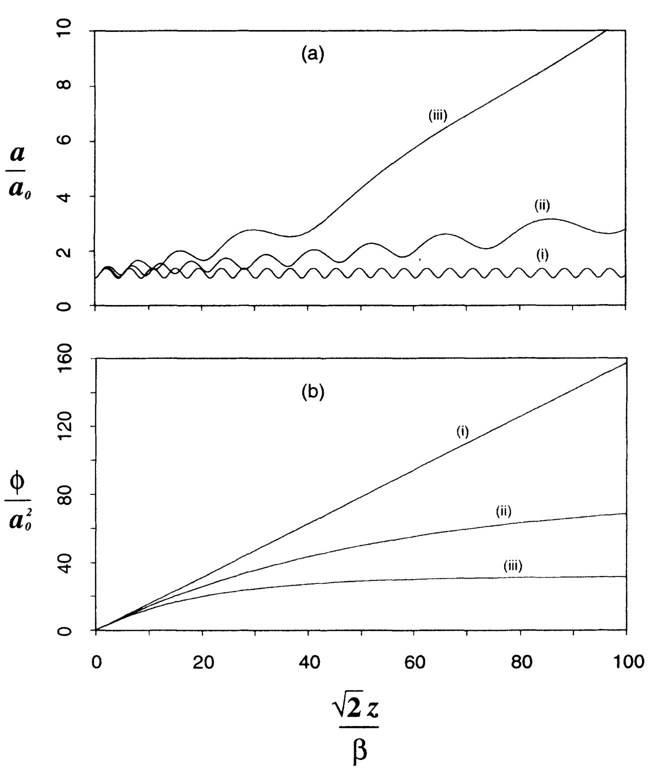

FIG. 4. Numerical solution for variation of normalized (a) spot width and (b) longitudinal phase with propagation in a lossy medium; $\mu+\lambda < \nu < 2(\mu+\lambda)$ with $\beta\gamma/\sqrt{2} =$ (i) 0.0, (ii) 0.01, (iii) 0.025. The initial beam width is a_0.

For the longitudinal phase, ϕ_z' cannot be considered as an eigenvalue of Eq. (4) for nonstationary propagation of cw Gaussian beams. Equation (11c), however, gives an insight into the interplay between diffraction, nonlinearity, and dissipation. From the numerical results, we note that ϕ_z' is largest for lossless propagation, and dissipation has the effect of reducing ϕ_z'. Moreover, ϕ_z' varies significantly as the beam begins to behave more as if it were a linear wave.

For the lossy case, we are not able to find the first integral of the governing equation for the spot width and hence cannot analyze the motion on the basis of a "potential well" description. However, we can use the adiabatic idea to help us visualize the dynamics of the beam as it propagates. We can rewrite Eq. (34) in the form

$$\beta^2 \frac{d^2 y}{dz^2} = \frac{2\tilde{\mu}}{y^3} - \frac{\tilde{\nu}}{y^2} + \frac{2\tilde{\lambda}}{y^3} , \tag{36}$$

with

$$\tilde{\mu} = \mu , \quad \tilde{\nu} = \nu e^{-2\gamma z} , \quad \tilde{\lambda} = \lambda e^{-4\gamma z} . \tag{37}$$

By making the parameter space transformation $(\mu,\nu,\lambda)\rightarrow(\tilde{\mu},\tilde{\nu},\tilde{\lambda})$, from Fig. 3 we see that as the beam propagates it is carried across different regions of the $(\tilde{\mu},\tilde{\nu},\tilde{\lambda})$ parameter space due to the dissipation, and is found to execute the motion in the corresponding "potential well" in which it finds itself. We note that the period z_p is increasing until reaching some critical distance z_{cr}, at which point the soliton property no longer holds. It is expected that $z_{cr} \gg z_p$. For $z \gg z_{cr}$, the wave behaves as though it was linear.

V. CONCLUSION

The propagation of cw Gaussian beams in a saturable medium with and without loss has been analyzed via a variational procedure. The dynamics of the beam can be described by a set of coupled ordinary differential equations. In the lossless case, we were able to find exact analytic solutions for the behavior of the spot width and were able to determine conditions under which steady-state ("solitonic") propagation was possible. Nonideal launching of a Gaussian gave rise to an oscillatory development of the spot width and longitudinal phase, and these results were compared to those for a Kerr-type nonlinearity. Gaussian beams under dissipation were also considered, and these results agreed with recent numerical simulations.

We have not explicitly considered the situation where $\lambda<0$. We note that in that case the higher-order nonlinearity would aid in the self-focusing of the cw Gaussian beams. The same type of behavior would be expected as with $\lambda>0$, but at different points on the (μ,ν,λ) parameter space.

In this paper, we have confined our attention to the propagation of beams in a medium with one transverse dimension. As is well known, a beam propagating in a Kerr medium with two transverse dimensions is unstable to symmetric perturbations. At a certain critical power, such beams undergo catastrophic collapse, while below the critical power they diffract. One way to overcome diffraction below the critical power is to use a graded refractive-index profile in the transverse directions to refocus the beam [26]. On the other hand, above the critical power, a saturable medium will support stable self-trapped beams. Karlsson [27] has shown that it is possible to realize beams that are stable to symmetric perturbations in a medium with saturation modeled as a two-level system. However, for this model, it is not possible to obtain analytic solutions.

We have investigated the propagation of Gaussian beams in a medium with two transverse dimensions and saturation modeled by Eq. (1), and have obtained exact analytic solutions for the spot width. These results will be presented elsewhere.

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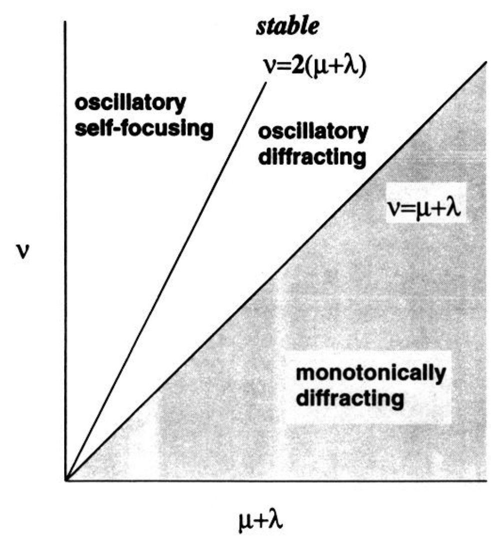

FIG. 3. Regions of stability and instability in the $(\nu, \mu+\lambda)$ plane.